QP CODE: 25050450

25050450

Reg No : .....

Name : .....

MSc DEGREE (CSS)EXAMINATION, DECEMBER 2025

First Semester

M Sc MATHEMATICS

Core Courses - ME010101 - ABSTRACT ALGEBRA

2019 Admission Onwards

5C247BAF

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. List all the elements of finite order of the group $\mathbb{Z}_2 \times \mathbb{Z} \times \mathbb{Z}_4$. Justify your answer.
2. Prove that if m is a square free integer then every abelian group of order m is cyclic.
3. Let H be a subgroup of a group G. Prove that G is an H - set under the action $*(h,g) = hg$ for $h \in H$ and $g \in G$.
4. Find the kernel of the homomorphism $\phi : \mathbb{Z}_{12} \to \mathbb{Z}_3$, where $\phi(1) = 2$.
5. Describe class equation of a finite group G.
6. Show that every group of order 45 has a normal subgroup of order 9.
7. Find $f(x)g(x)$ in $\mathbb{Z}_7$, where $f(x) = x^3 + 2x^2 + 5$ and $g(x) = 3x^2 + 2x$
8. Check $8x^3 + 6x^2 - 9x + 24$ is irreducible over $\mathbb{Q}$
9. Define Quarternion ring.
10. Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$ and let N be the subring of all functions f such that f(2) = 0. Is N an ideal in F? Why or Why not?

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let H be a normal subgroup of a group G. Prove that the cosets of H form a group G/H under the binary operation $(aH)(bH) = (ab)H$

12. Let G be a finite group and X a finite G – set. If r is the number of orbits in X under G, prove that $r \cdot |G| = \sum_{g \in G} |X_g|$

13. Let G be a finite group and let $|G| = p^n m$, where $n \geq 1$ and p does not divide m. Prove that G contains a subgroup of order p^i, for each i, where $1 \leq i \leq n$.

14. Prove that no group of order 36 is simple.

15. Show that for every integer n, the number $n^{33} - n$ is divisible by 15

16. Show that for [(a,b)] and[(c,d)] in F the equations [(a,b)] +[(c,d)] = [(ad+bc,bd)] and [(a,b)] [(c,d)] = [(ac,bd)] give well defined addition and multiplication on F where F be the desired field of quotients of an integral domain D.

17. Show that if R, R' and R'' are rings and if $\phi : R \to R'$ and $\psi : R' \to R''$ are homomorphisms, then the composite function $\psi : R' \to R''$ is a homomorphism.

18. If R is a ring with unity 1, then show that the map $\phi : \mathbb{Z} \to R$ given by $\phi(n) = n \cdot 1$ for $n \in \mathbb{Z}$ is a homomorphism of $\mathbb{Z}$ into R.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) Let X be a G – set. Prove that $\{g \in G / gx = x\}$ is a subgroup of G for each $x \in X$.
(b) Let X be a G – set and let $x \in X$. Prove that $|Gx| = (G : G_x)$.

20. (a) Let p be a prime. Let G be a finite group and let p divide |G|. Prove that G has a subgroup of order p.
(b) Let G be a finite group. Prove that G is a p-group if and only if |G| is a power of p.

21. (a) Explain the division algorithm in F[x].
(b) If G is a finite subgroup of the multiplicative group of a field F, then prove that G is cyclic.

22. Define
(a) Endomorphism ring
(b) Group ring
(c) Ring homomorphism
(d) Ideal of a ring
(e) Factor ring
(f) Prime ideal
(g) Maximal ideal
(h) Principal ideal

(2×5=10 weightage)